 

# Exam II: MTH 111, Fall 2017

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Points = $\frac{47}{47}$

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QUESTION 1. (8 points) Find $y'$ and DO NOT SIMPLIFY

(i) $y = 2x^3 + 10x - 7$

$y' = 6x^2 + 10.$

(ii) $y = \sqrt{x} + (3x-1)^{11}$ $\rightarrow y = x^{1/2} + (3x-1)^{11}$

$y' = \frac{1}{2\sqrt{x}} + 11(3x-1)^{10}(3).$

$y' = \frac{1}{2\sqrt{x}} + 33(3x-1)^{10}.$

(iii) $y = \frac{4}{x^2} + \frac{2}{\sqrt{x^2+4}}$

$y = 4x^{-2} + 2(x^2+4)^{-1/2}$

$y' = -8x^{-3} + \cancel{2}\left(-\frac{1}{\cancel{2}}\right)(x^2+4)^{-3/2}(2x)$

$y' = -\frac{8}{x^3} - 2x(x^2+4)^{-3/2}.$

(iv) Given $y = k(4x^2 - x)$ such that $k'(3) = -7$. Find $y'(1)$ (i.e., evaluate $y'$ when $x = 1$.)

$y' = (8x-1)k'(4x^2-x)$

$y'|_{x=1} = 7k'(3) = 7(-7) = -49.$

QUESTION 2. (i) (3 points) Can we draw the vector $v = < 3, -5, 2 >$ inside the plane $x - 4y - 11z = 7$? explain

$V = <3, -5, 2>$

$N = <1, -4, -11>$

$N \cdot V = 3(1) - 5(-4) + 2(-11)$

$N \cdot V = 3 + 20 - 22 = 1 \neq 0$

NO. The two vectors are not perpendicular, hence V can't be drawn inside the plane.

(ii) (4 points) Given $N = < 4, 6, 2 >$ is perpendicular to the plane $P$ and the point $(4, 1, 1)$ lies inside the plane $P$. Find the equation of the plane $P$.

$N = <4, 6, 2>$
$<a, b, c>$
$Q(4, 1, 1)$
$Q(x_0, y_0, z_0)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$4(x-4) + 6(y-1) + 2(z-1) = 0$

$4x - 16 + 6y - 6 + 2z - 2 = 0$

$\boxed{4x + 6y + 2z = 24}$

(iii) (6 points)Find the equation of the plane that contains the points $Q_1 = (1, 1, 4)$, $Q_2 = (2, 3, 6)$ and $Q_3 = (1, 1, 8)$.

$Q_1(1, 1, 4)$
$Q_2(2, 3, 6)$
$Q_3(1, 1, 8)$

$\overrightarrow{Q_1Q_2} = <1, 2, 2>$

$\overrightarrow{Q_1Q_3} = <0, 0, 4>$

$\vec{N} = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$\vec{N} = 8\hat{i} - 4\hat{j} + 0\hat{k}$

$\boxed{\vec{N} = <8, -4, 0>}$

~~$8(x-1) - 4(y+4) + 0(z)$~~

$8(x-1) - 4(y-1) + 0(z-4) = 0$

$8x - 8 - 4y + 4 = 0$

$\boxed{8x - 4y = 4}$

$\boxed{2x - y = 1}$

QUESTION 3. (i) **(4 points)** The line $L : x = 2w, y = -w + 1, z = 3$ intersects the plane $4x + 7y + z = 12$ in a point $Q$. Find $Q$.

$$L: \begin{cases} x = 2w \\ y = -w+1 \; ; \; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: \; 4x + 7y + z = 12$

$4(2w) + 7(-w+1) + 3 = 12$

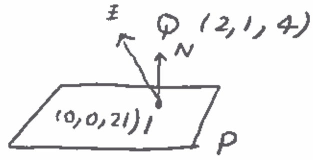
$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ → The plane and the line intersect when $w = 2$

$\Rightarrow \boxed{Q(4, -1, 3)}$

(ii) **(4 points)** Find the distance between $Q = (2, 1, 4)$ and the plane $2x - 2y + z = 21$.




$I\,(0, 0, 21)$

$Q\;(2, 1, 4)$

$\vec{IQ} = \langle 2, 1, -17 \rangle$

$N = \langle 2, -2, 1 \rangle$

$$d = \frac{|\vec{IQ} \cdot N|}{|N|} = \frac{|2(2) + 1(-2) + 1(-17)|}{\sqrt{4 + 4 + 1}}$$

$$d = \frac{15}{\sqrt{9}} = \frac{15}{3} = 5 \text{ units}$$

(iii) **(6 points)** The two planes $P_1 : x + y + z = 2$ and $P_2 : -x + y - z = 6$ intersects in a line $L$. Find a parametric equations of $L$.

$N_1 = \langle 1, 1, 1 \rangle$

$N_2 = \langle -1, 1, -1 \rangle$

$$\vec{D} = \vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix}$$

$$\vec{D} = -2\hat{i} + 0\hat{j} + 2\hat{k}$$

→ Let $z = 0$ ; find $x$ and $y$ :

$$\begin{cases} x + y = 2 \\ -x + y = 6 \end{cases}$$

$2y = 8$

$\boxed{y = 4}$

$x + 4 = 2$

$x = 2 - 4$

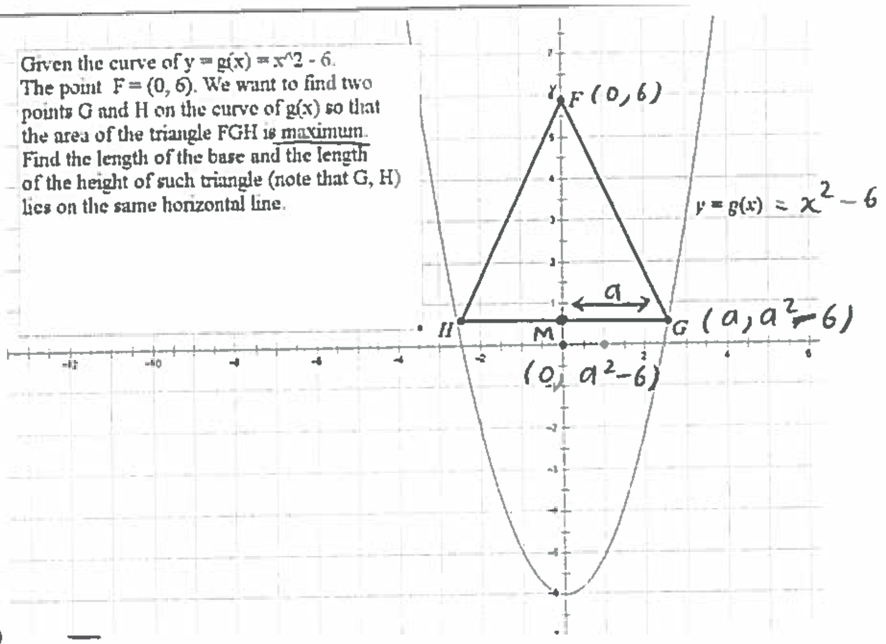
$\boxed{x = -2}$

The point is $(-2, 4, 0)$ and $D = \langle -2, 0, 2 \rangle$

* Parametric Eqns :

$$L: \begin{cases} x = -2 - 2t \\ y = 4 \\ z = 2t \end{cases} ; \; t \in \mathbb{R}$$

Given the curve of y = g(x) = x^2 - 6.
The point F = (0, 6). We want to find two points G and H on the curve of g(x) so that the area of the triangle FGH is maximum.
Find the length of the base and the length of the height of such triangle (note that G, H) lies on the same horizontal line.




**QUESTION 4. (6 points)**

Base $= \overline{GH} = 2a$ ; where $a > 0$.

Height $= \overline{FM} = 6 - (a^2 - 6)$

$\overline{FM} = 6 + 6 - a^2 = 12 - a^2$

$A\triangle = \frac{1}{2} bh$

$A\triangle = \frac{1}{2}(2a)(12 - a^2) = 12a - a^3$

$A' = 12 - 3a^2$

$A' = 0$

$12 = 3a^2$

$a^2 = 4 \rightarrow a = \pm 2$ because $a > 0$.

$\boxed{a = +2}$

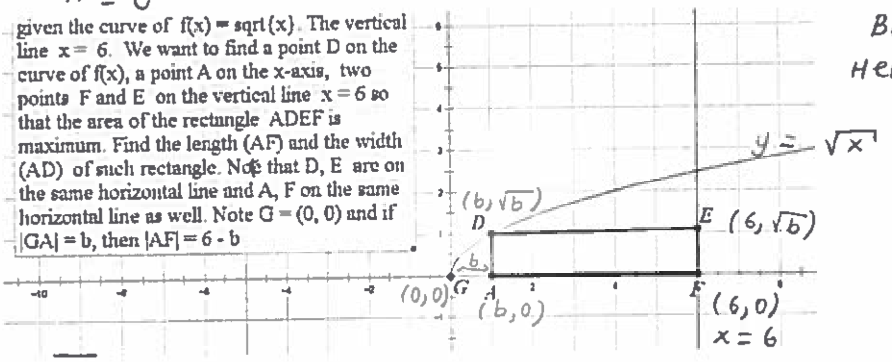
$A'' = -6a$

$A''|_{a=2} = -12 < 0 \rightarrow$ curve MAXIMU when $\boxed{a = 2}$

Base $= \overline{GH} = 2(2) = 4$

Height $= \overline{FM} = 12 - 4 = 8$

given the curve of f(x) = sqrt(x). The vertical line x = 6. We want to find a point D on the curve of f(x), a point A on the x-axis, two points F and E on the vertical line x = 6 so that the area of the rectangle ADEF is maximum. Find the length (AF) and the width (AD) of such rectangle. Note that D, E are on the same horizontal line and A, F on the same horizontal line as well. Note G = (0, 0) and if |GA| = b, then |AF| = 6 - b




A ($b$,0)
F (6,0)

D ($b, \sqrt{b}$)
E ($6, \sqrt{b}$)

**QUESTION 5. (6 points)**

width $= \overline{AF} = 6 - b$

$\overline{AD} =$ ~~height~~ length $= \sqrt{b}$

$A\square = l \times w$

$A\square = \sqrt{b}(6 - b) = b^{1/2}(6 - b)$

$A\square = 6b^{1/2} - b^{3/2}$

$A' = \frac{6}{2\sqrt{b}} - \frac{3\sqrt{b}}{2}$

$A' = \frac{3}{\sqrt{b}} - \frac{3\sqrt{b}}{2}$

$A' = \frac{(2)3 - 3b}{2\sqrt{b}}$

$A' = \frac{6 - 3b}{2\sqrt{b}}$

$A' = 0$.

$6 - 3b = 0$

$3b = 6$

$\boxed{b = 2}$

CHECK $A''$:

$A'' = \frac{-3(b^{-3/2})}{2} - \frac{3}{4\sqrt{b}}$

$A''|_{b=2} < 0$ ∩ MAX. when $\boxed{b = 2}$

$\overline{AF}$ = width $= 6 - 2 = 4$.

$\overline{AD}$ = length $= \sqrt{2}$.

**Faculty information**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Side Note:

| $x$ | | $+2$ | |
|---|---|---|---|
| $f'(x)$ | + | 0 | − |
| $f(x)$ | ↗ | | ↘ |

The curve Max

* Area $\square = 4\sqrt{2}$ units$^2$.